Computer Organization and Architecture

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iClicker

How many minimum number of bits do we need to identify 6 types of bagels uniquely?

A 2

B 3

C 4

D 6

E 64

Truth Table

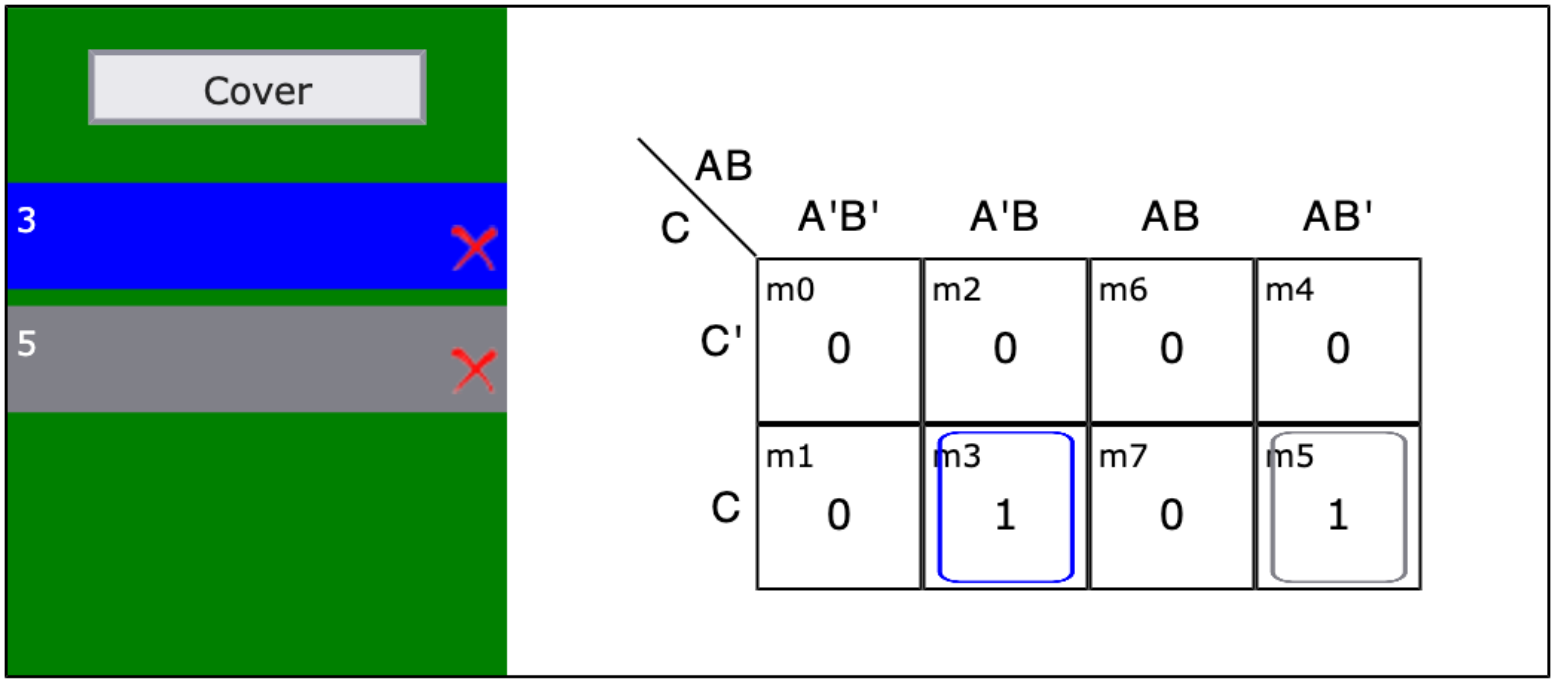
What's your favorite ?

Bagels	A	B	C	F(A, B, C)
Plain	0	0	0	0
Everything	0	0	1	0
Sesame	0	1	0	0
Cinnamon	0	1	1	1
Raisin	1	0	0	0
Jalapeno-Cheddar	1	0	1	1

Truth Table

Bagels	A	B	C	F(A, B, C)
Plain	0	0	0	0
Everything	0	0	1	0
Sesame	0	1	0	0
Cinnamon	0	1	1	1
Raisin	1	0	0	0
Jalapeno-Cheddar	1	0	1	1
	1	1	0	?
	1	1	1	?

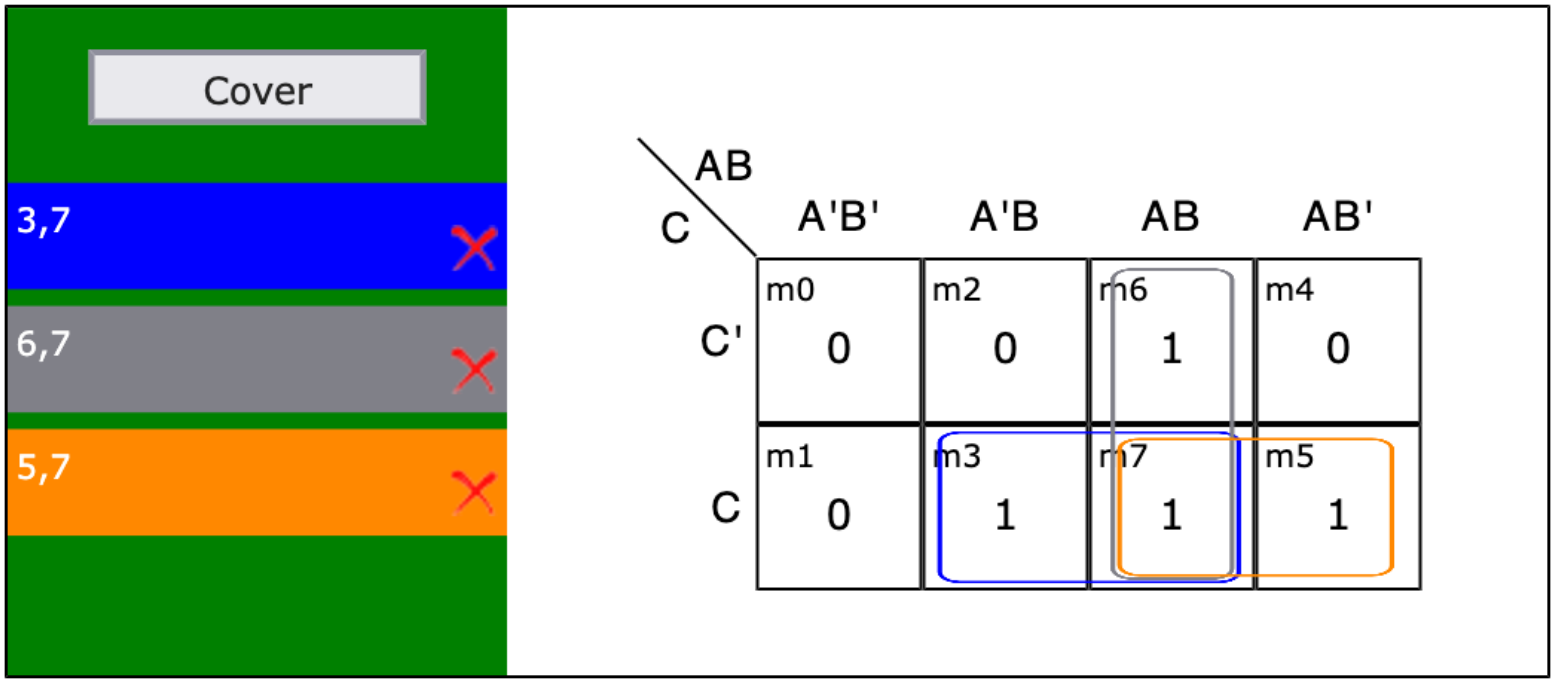
Truth Table



+ AB'C

A'BC

Truth Table



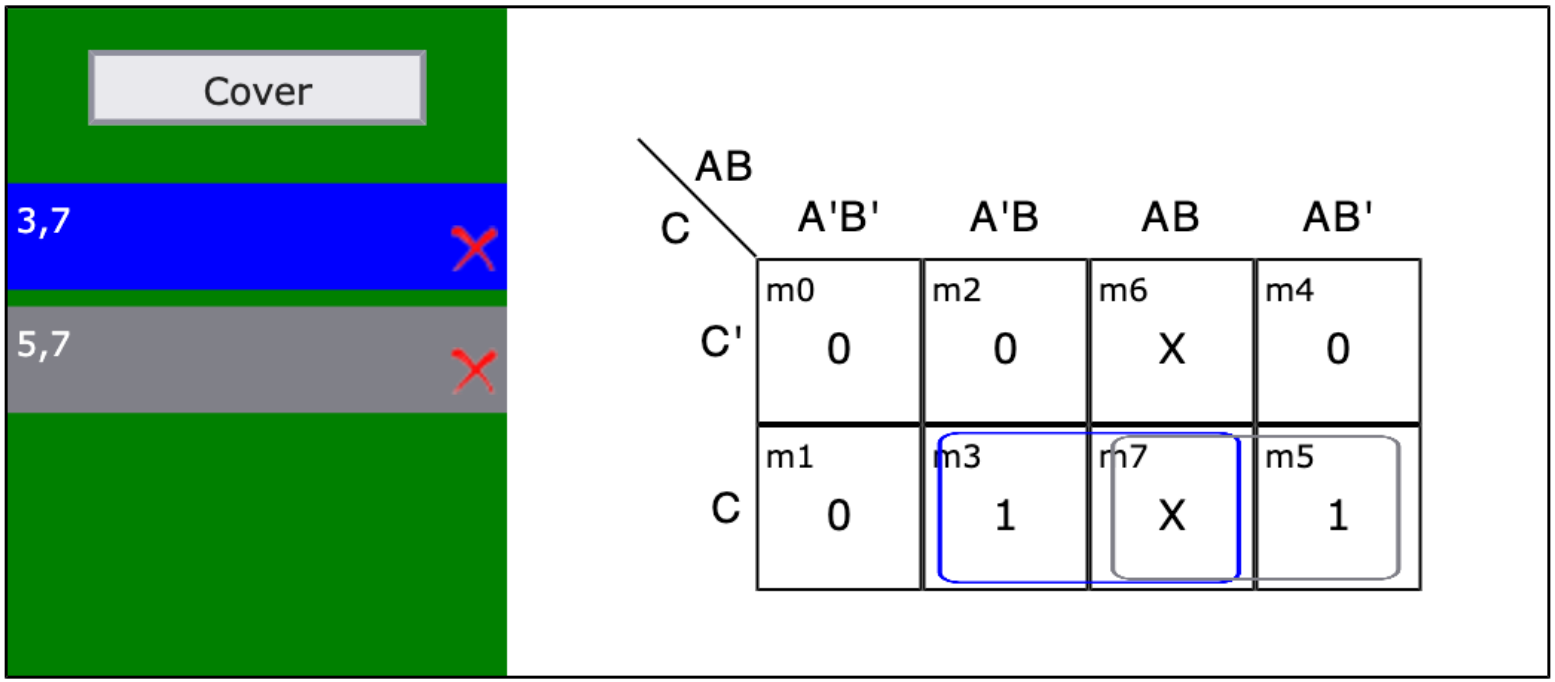
+ BC + AC

AB

Truth Table

Bagels	A	B	C	F(A, B, C)
Plain	0	0	0	0
Everything	0	0	1	0
Sesame	0	1	0	0
Cinnamon	0	1	1	1
Raisin	1	0	0	0
Jalapeno-Cheddar	1	0	1	1
	1	1	0	X
	1	1	1	X

Truth Table



+ BC

AC

K-Map: 3 variables

F(A, B, C) = m1, m3, m4, m5 and
don't care = m0, m7

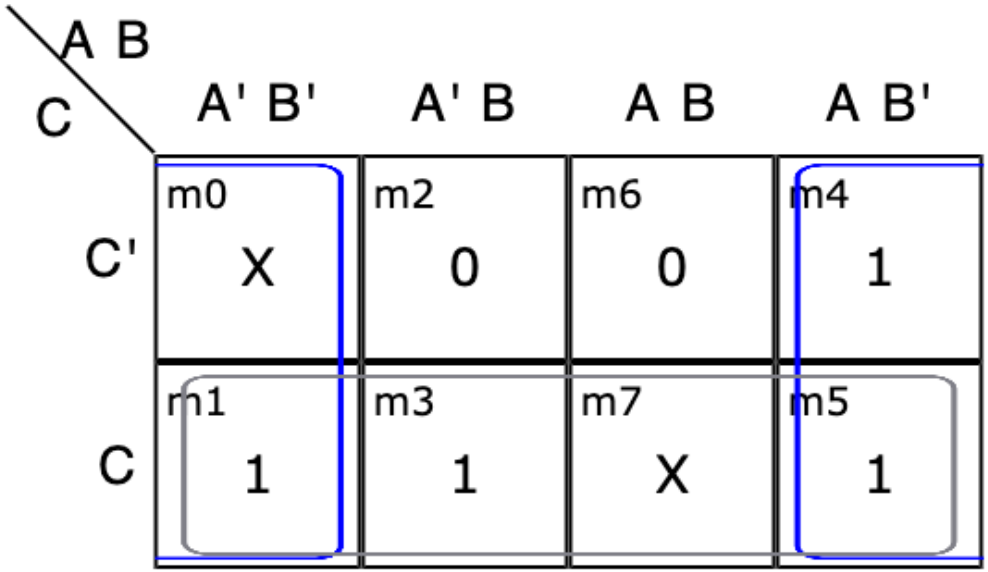
K-Map: 3 variables

F(A, B, C) = m1, m3, m4, m5 and
don't care = m0, m7

C \ A B	A' B'	A' B	A B	A B'
C'	m0 X	m2 0	m6 0	m4 1
C	m1 1	m3 1	m7 X	m5 1

K-Map: 3 variables

F(A, B, C) = m1, m3, m4, m5 and
don't care = m0, m7



F(A, B, C) = B' + C

K-Map: 3 variables

F(A, B, C) = m4, m5, m6, m7 and
don't care = m0, m2

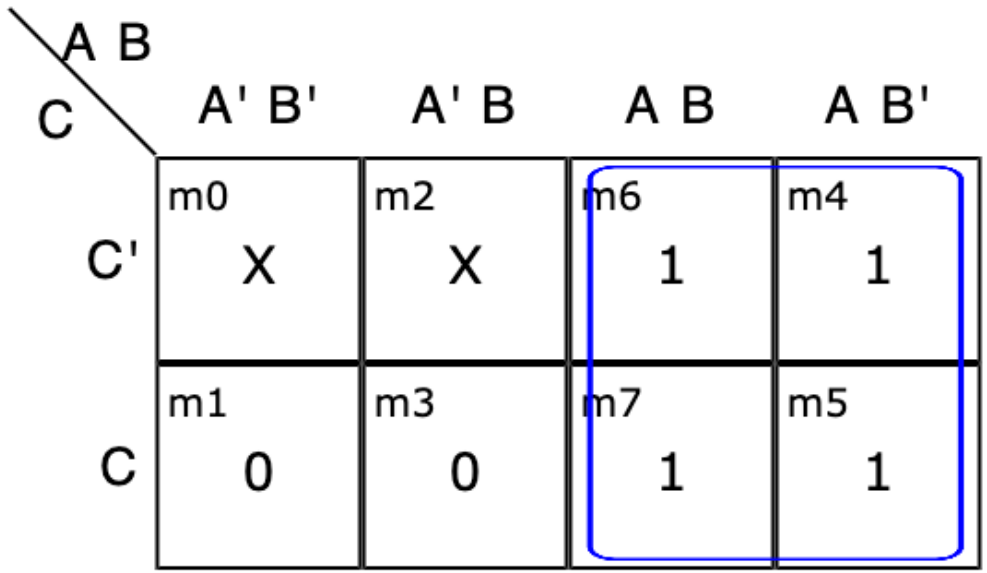
K-Map: 3 variables

F(A, B, C) = m4, m5, m6, m7 and
don't care = m0, m2

C \ A B	A' B'	A' B	A B	A B'
C'	m0 X	m2 X	m6 1	m4 1
C	m1 0	m3 0	m7 1	m5 1

K-Map: 3 variables

F(A, B, C) = m4, m5, m6, m7 and
don't care = m0, m2



F(A, B, C) = A

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F(A, B, C) = m3, m7

Don't care = m5, m6

C \ A B	A' B'	A' B	A B	A B'
C'	m0 0	m2 0	m6 X	m4 0
C	m1 0	m3 1	m7 1	m5 X

K-Map: 4 variables

F(A, B, C, D) = m2, m4, m6, m8, m11, m12, m13, m15
Don't care = m0, m9, m10

K-Map: 4 variables

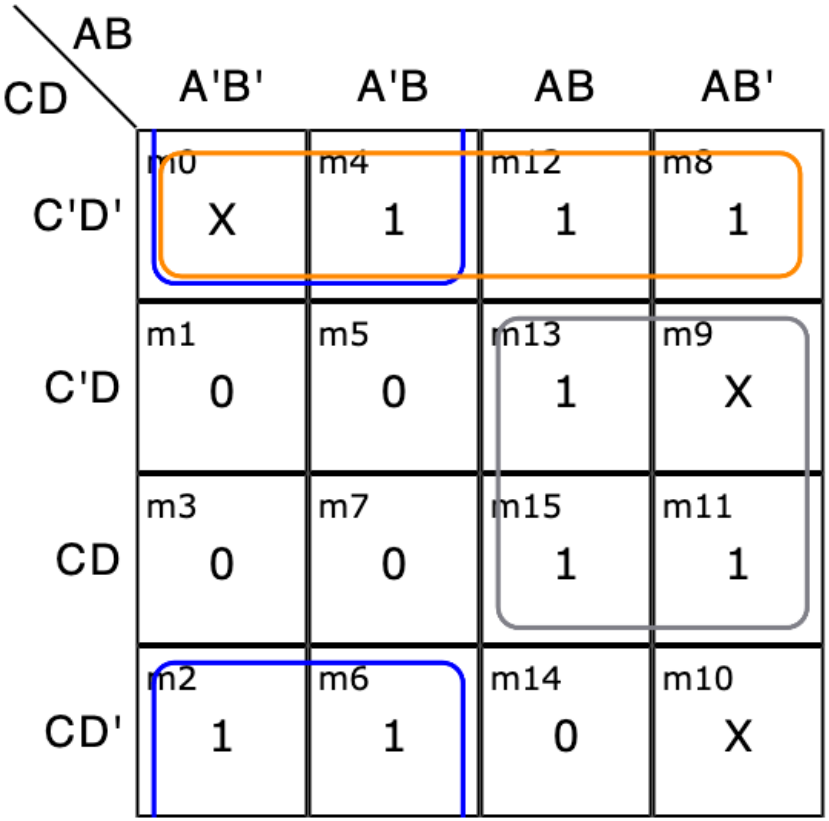
F(A, B, C, D) = m2, m4, m6, m8, m11, m12, m13, m15
Don't care = m0, m9, m10

CD \ AB	A'B'	A'B	AB	AB'
C'D'	m0 X	m4 1	m12 1	m8 1
C'D	m1 0	m5 0	m13 1	m9 X
CD	m3 0	m7 0	m15 1	m11 1
CD'	m2 1	m6 1	m14 0	m10 X

K-Map: 4 variables

F(A, B, C, D) = m2, m4, m6, m8, m11, m12, m13, m15

Don't care = m0, m9, m10



F(A, B, C, D) = A'D' + AD + C'D'

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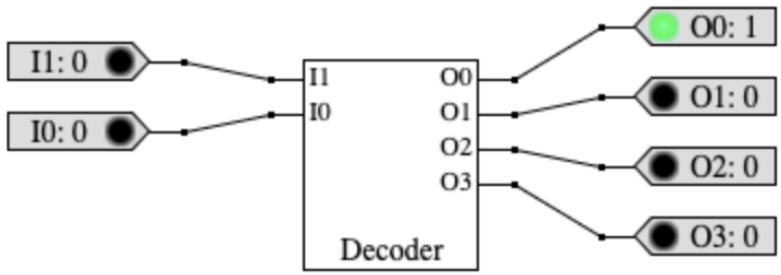
F(A, B, C, D) = m0, m1, m2, m3, m5, m12, m13, m14

Don't care = m4, m8, m9, m10, m15

CD \ AB	A'B'	A'B	AB	AB'
C'D'	m0 1	m4 X	m12 1	m8 X
C'D	m1 1	m5 1	m13 1	m9 X
CD	m3 1	m7 0	m15 X	m11 0
CD'	m2 1	m6 0	m14 1	m10 X

Decoder

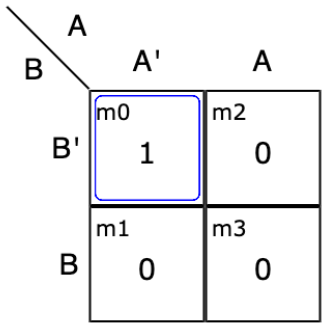
- A decoder recognizes which encoded value is the input to the circuit
- Input: n
- Output: 2^n
- One of the 2^n outputs is asserted, based on which encoded value is input.



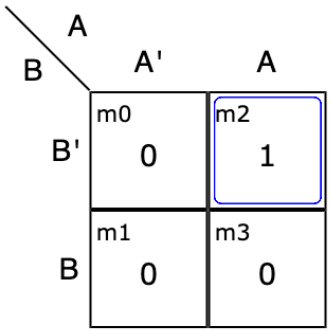
Decoder

I_1	I_0	O_3	O_2	O_1	O_0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

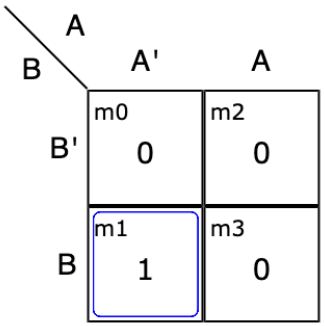
Decoder



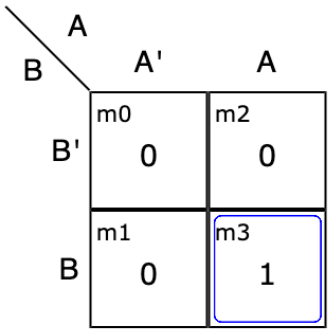
O_0 = A'B' = $I_1' I_0'$



O_2 = AB' = $I_1 I_0'$

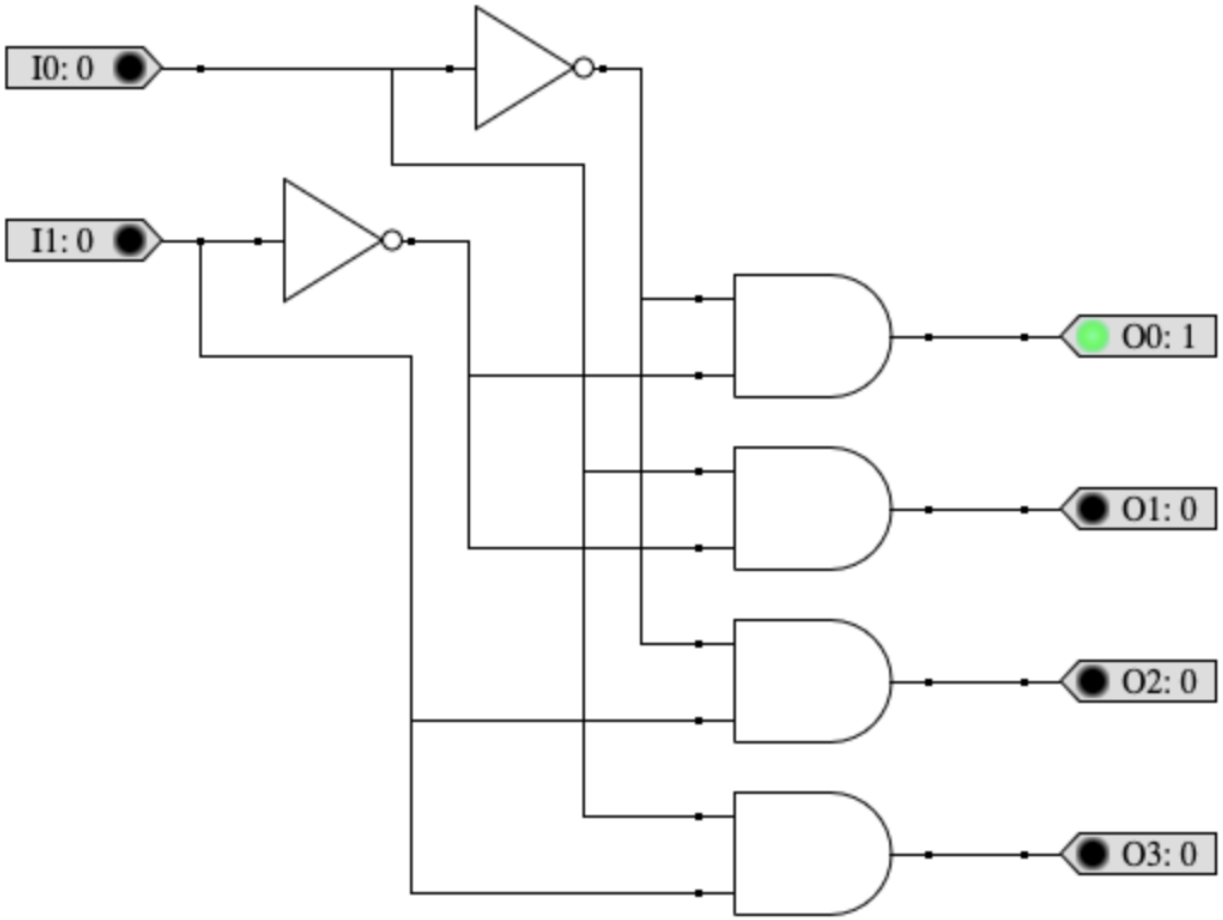


O_1 = A'B = $I_1' I_0$



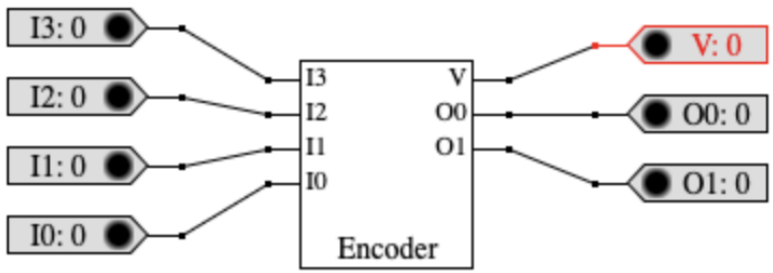
O_3 = AB = $I_1 I_0$

Decoder



Encoder

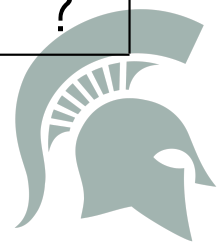
- Input: 2^n
- Output: n



Encoder

I_3	I_2	I_1	I_0		O_1	O_0
0	0	0	0		?	?
0	0	0	1		?	?
0	0	1	0		?	?
0	0	1	1		?	?
0	1	0	0		?	?
0	1	0	1		?	?
0	1	1	0		?	?
0	1	1	1		?	?

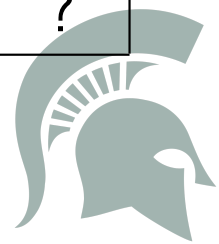
I_3	I_2	I_1	I_0		O_1	O_0
1	0	0	0		?	?
1	0	0	1		?	?
1	0	1	0		?	?
1	0	1	1		?	?
1	1	0	0		?	?
1	1	0	1		?	?
1	1	1	0		?	?
1	1	1	1		?	?



Encoder

I_3	I_2	I_1	I_0		O_1	O_0
0	0	0	0		?	?
0	0	0	1		0	0
0	0	1	0		0	1
0	0	1	1		?	?
0	1	0	0		1	0
0	1	0	1		?	?
0	1	1	0		?	?
0	1	1	1		?	?

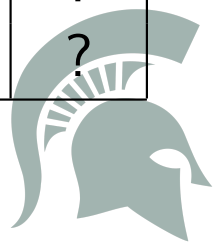
I_3	I_2	I_1	I_0		O_1	O_0
1	0	0	0		1	1
1	0	0	1		?	?
1	0	1	0		?	?
1	0	1	1		?	?
1	1	0	0		?	?
1	1	0	1		?	?
1	1	1	0		?	?
1	1	1	1		?	?



Encoder

I_3	I_2	I_1	I_0	V	O_1	O_0
0	0	0	0	0	?	?
0	0	0	1	1	0	0
0	0	1	0	1	0	1
0	0	1	1	0	?	?
0	1	0	0	1	1	0
0	1	0	1	0	?	?
0	1	1	0	0	?	?
0	1	1	1	0	?	?

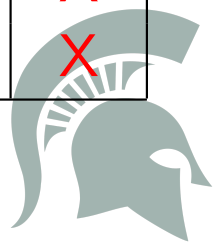
I_3	I_2	I_1	I_0	V	O_1	O_0
1	0	0	0	1	1	1
1	0	0	1	0	?	?
1	0	1	0	0	?	?
1	0	1	1	0	?	?
1	1	0	0	0	?	?
1	1	0	1	0	?	?
1	1	1	0	0	?	?
1	1	1	1	0	?	?



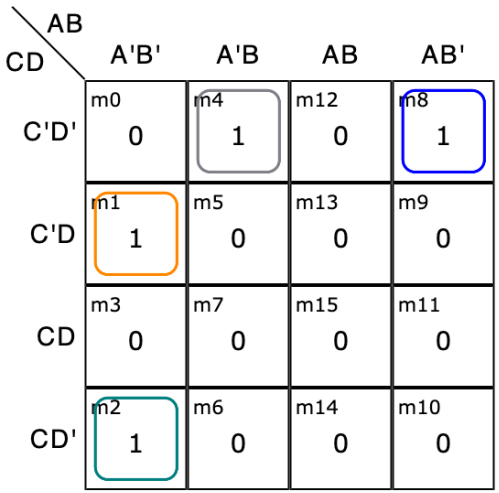
Encoder

I_3	I_2	I_1	I_0	V	O_1	O_0
0	0	0	0	0	X	X
0	0	0	1	1	0	0
0	0	1	0	1	0	1
0	0	1	1	0	X	X
0	1	0	0	1	1	0
0	1	0	1	0	X	X
0	1	1	0	0	X	X
0	1	1	1	0	X	X

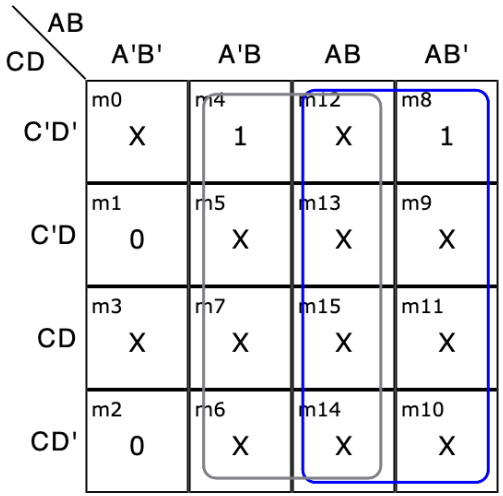
I_3	I_2	I_1	I_0	V	O_1	O_0
1	0	0	0	1	1	1
1	0	0	1	0	X	X
1	0	1	0	0	X	X
1	0	1	1	0	X	X
1	1	0	0	0	X	X
1	1	0	1	0	X	X
1	1	1	0	0	X	X
1	1	1	1	0	X	X



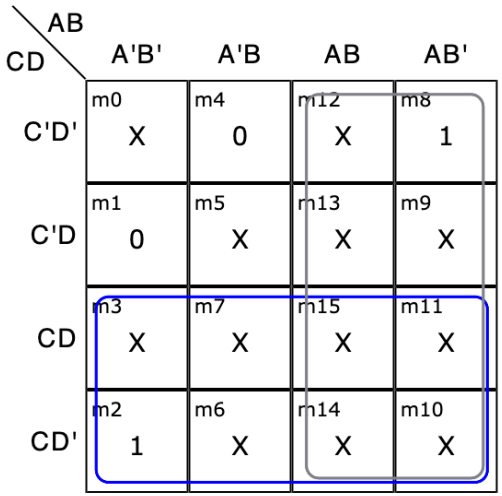
Encoder



$$V = AB'C'D' + A'BC'D' + A'B'CD' + A'B'C'D =$$
$$I_3 I_2' I_1' I_0' + I_3' I_2 I_1' I_0' + I_3' I_2' I_1 I_0' + I_3' I_2' I_1' I_0$$

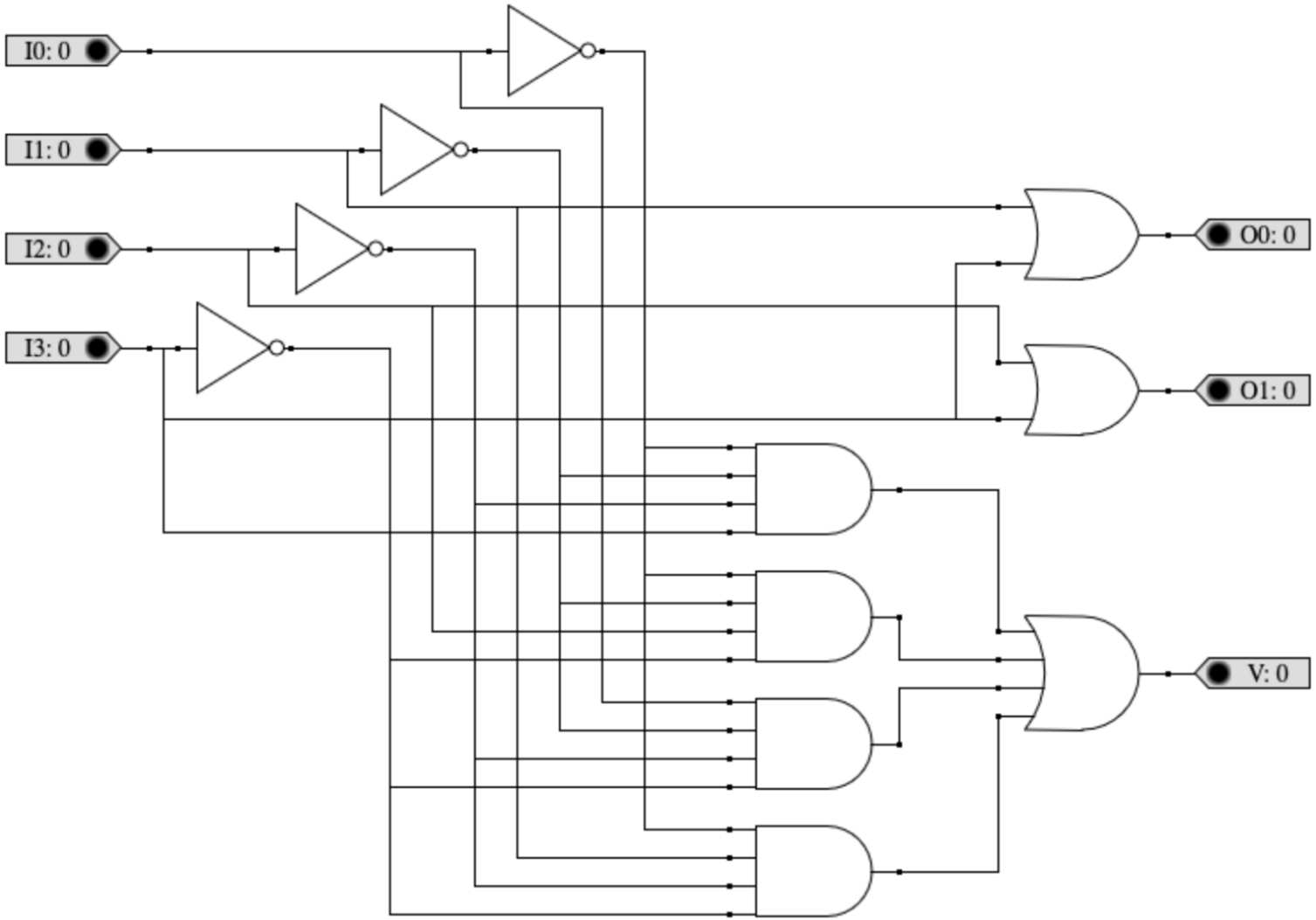


$$O_1 = A + B = I_3 + I_2$$



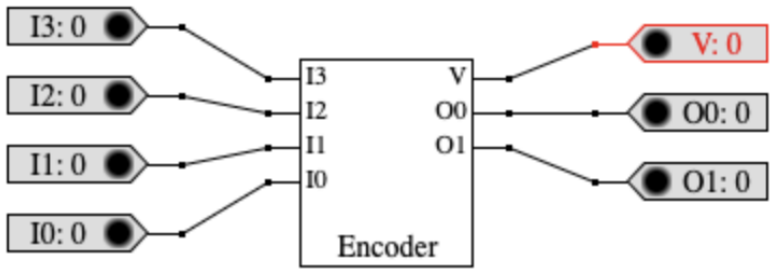
$$O_0 = A + C = I_3 + I_1$$

Encoder



Priority Encoder

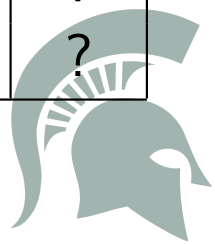
- Piority Encoder acts like an encoder, but there is a priority established to handle more than one input being asserted
- Input: 2^n
- Output: n
- One or more of the 2^n inputs is asserted, highest priority value is encoded
- Priority Order: $I_3 > I_2 > I_1 > I_0$



Priority Encoder

I_3	I_2	I_1	I_0	V	O_1	O_0
0	0	0	0		?	?
0	0	0	1	1	0	0
0	0	1	0	1	0	1
0	0	1	1		?	?
0	1	0	0	1	1	0
0	1	0	1		?	?
0	1	1	0		?	?
0	1	1	1		?	?

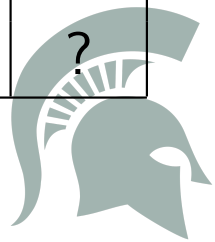
I_3	I_2	I_1	I_0	V	O_1	O_0
1	0	0	0	1	1	1
1	0	0	1		?	?
1	0	1	0		?	?
1	0	1	1		?	?
1	1	0	0		?	?
1	1	0	1		?	?
1	1	1	0		?	?
1	1	1	1		?	?



Priority Encoder

I_3	I_2	I_1	I_0	V	O_1	O_0
0	0	0	0		?	?
0	0	0	1	1	0	0
0	0	1	0	1	0	1
0	0	1	1	1	0	1
0	1	0	0	1	1	0
0	1	0	1		?	?
0	1	1	0		?	?
0	1	1	1		?	?

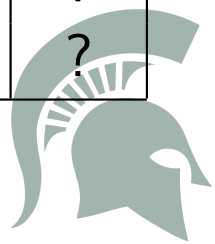
I_3	I_2	I_1	I_0	V	O_1	O_0
1	0	0	0	1	1	1
1	0	0	1		?	?
1	0	1	0		?	?
1	0	1	1		?	?
1	1	0	0		?	?
1	1	0	1		?	?
1	1	1	0		?	?
1	1	1	1		?	?



Priority Encoder

I_3	I_2	I_1	I_0	V	O_1	O_0
0	0	0	0		?	?
0	0	0	1	1	0	0
0	0	1	0	1	0	1
0	0	1	1	1	0	1
0	1	0	0	1	1	0
0	1	0	1	1	1	0
0	1	1	0	1	1	0
0	1	1	1	1	1	0

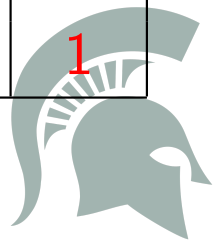
I_3	I_2	I_1	I_0	V	O_1	O_0
1	0	0	0	1	1	1
1	0	0	1		?	?
1	0	1	0		?	?
1	0	1	1		?	?
1	1	0	0		?	?
1	1	0	1		?	?
1	1	1	0		?	?
1	1	1	1		?	?



Priority Encoder

I_3	I_2	I_1	I_0	V	O_1	O_0
0	0	0	0		?	?
0	0	0	1	1	0	0
0	0	1	0	1	0	1
0	0	1	1	1	0	1
0	1	0	0	1	1	0
0	1	0	1	1	1	0
0	1	1	0	1	1	0
0	1	1	1	1	1	0

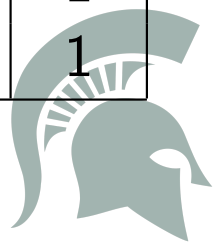
I_3	I_2	I_1	I_0	V	O_1	O_0
1	0	0	0	1	1	1
1	0	0	1	1	1	1
1	0	1	0	1	1	1
1	0	1	1	1	1	1
1	1	0	0	1	1	1
1	1	0	1	1	1	1
1	1	1	0	1	1	1
1	1	1	1	1	1	1



Priority Encoder

I_3	I_2	I_1	I_0	V	O_1	O_0
0	0	0	0	0	X	X
0	0	0	1	1	0	0
0	0	1	0	1	0	1
0	0	1	1	1	0	1
0	1	0	0	1	1	0
0	1	0	1	1	1	0
0	1	1	0	1	1	0
0	1	1	1	1	1	0

I_3	I_2	I_1	I_0	V	O_1	O_0
1	0	0	0	1	1	1
1	0	0	1	1	1	1
1	0	1	0	1	1	1
1	0	1	1	1	1	1
1	1	0	0	1	1	1
1	1	0	1	1	1	1
1	1	1	0	1	1	1
1	1	1	1	1	1	1

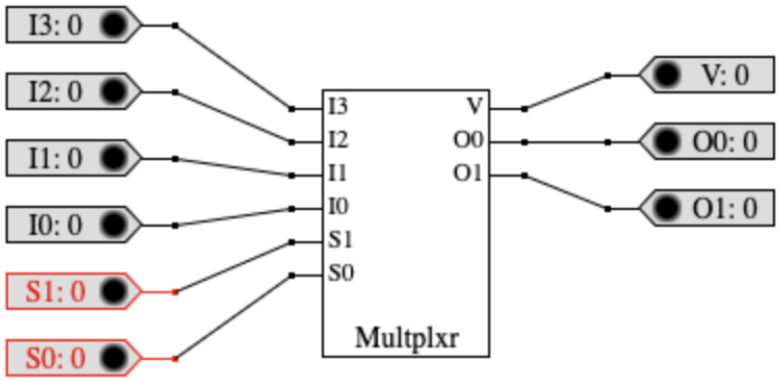


Priority Encoder

Expression and Circuit: Quiz !

Multiplexer

- A multiplexer is used to select one input (out of several inputs) to be the circuit output
- Input: 2^n
- Selection input: n
- Output: 1
- The n selection bits determine which of the 2^n data inputs is routed to the output



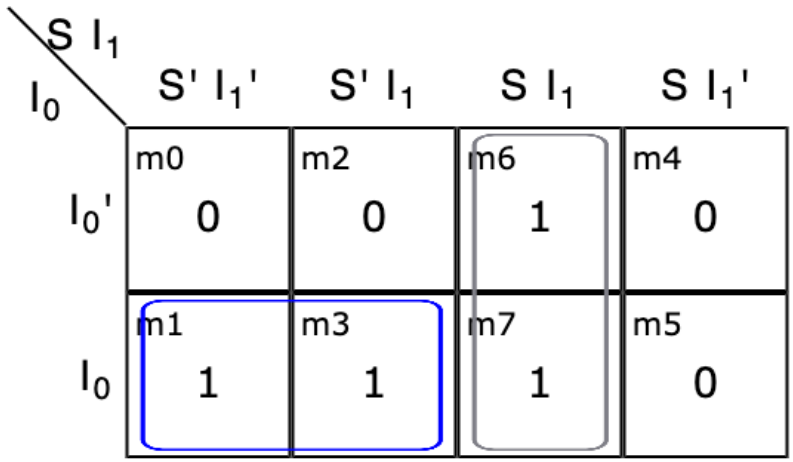
2-to-1 Multiplexer

S	O
0	I_0
1	I_1

S	I_1	I_0	O
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

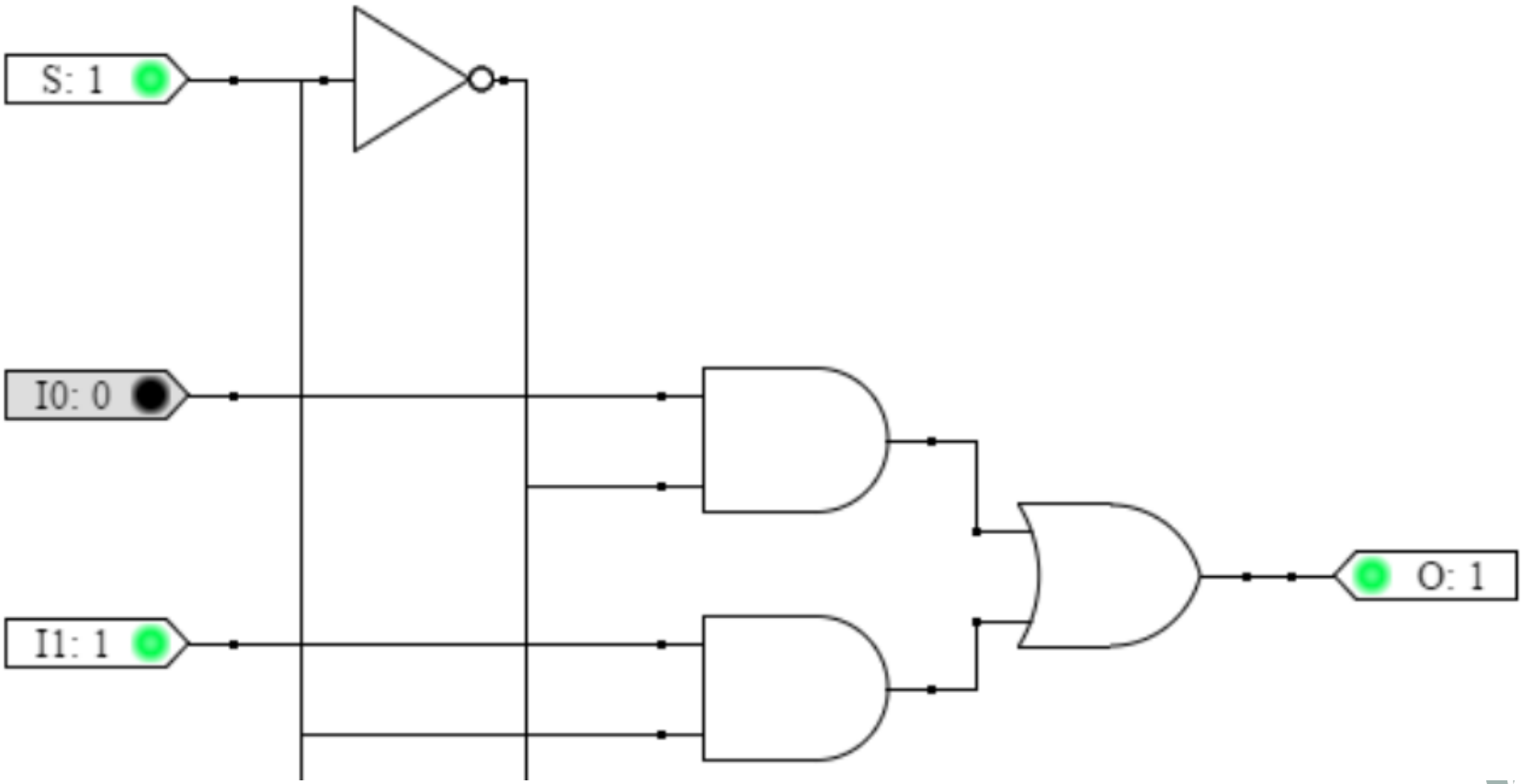
2-to-1 Multiplexer

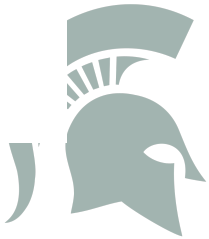
S	I_1	I_0	O
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1



$SI_1 + S'I_0$

2-to-1 Multiplexer





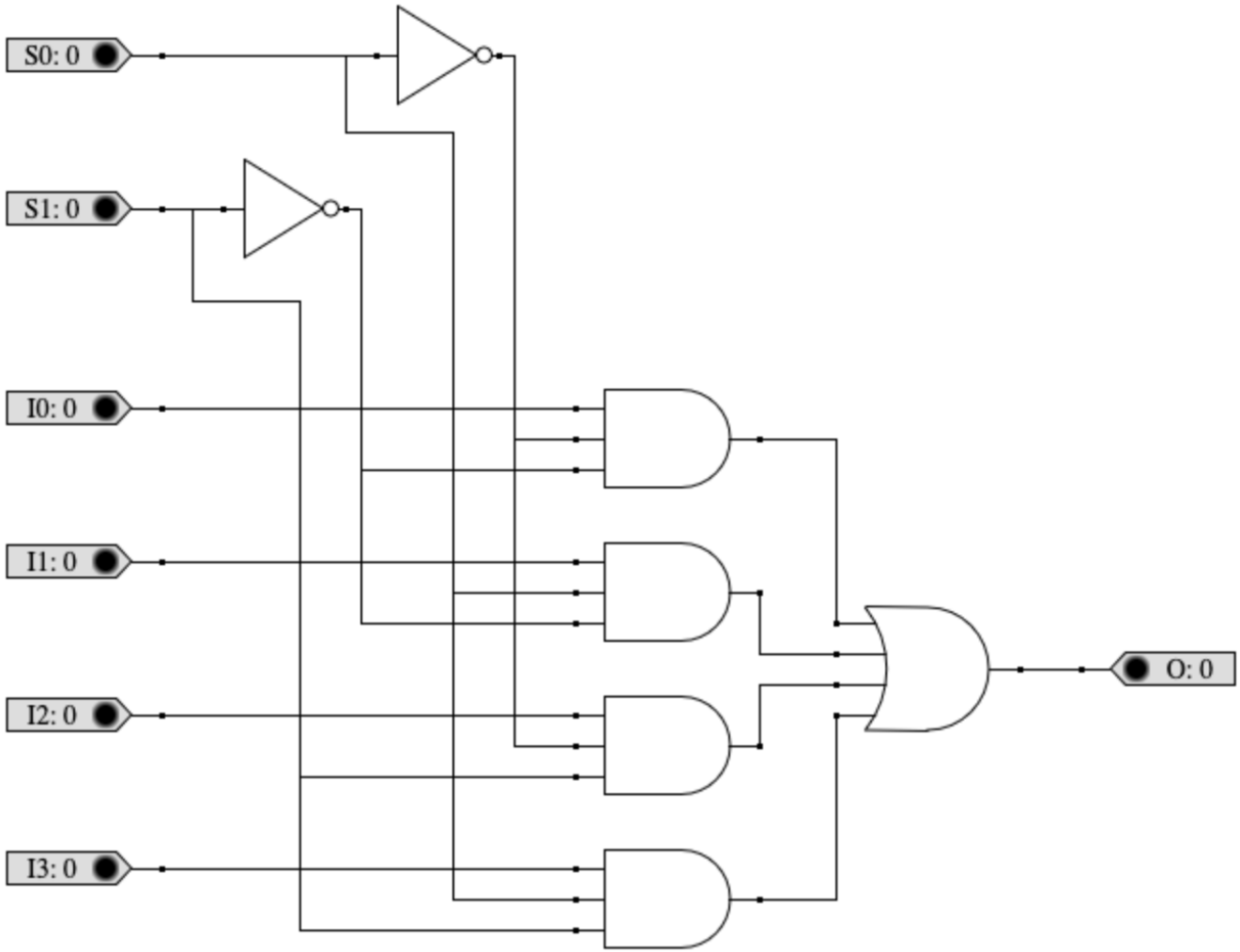
4-to-1 Multiplexer

S_1	S_0	O
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

4-to-1 Multiplexer

6-inputs, so the characteristic table will contain $2^6 = 64$ entries

4-to-1 Multiplexer



Demultiplexer

S_1	S_0	O_3	O_2	O_1	O_0
0	0	0	0	0	I
0	1	0	0	I	0
1	0	0	I	0	0
1	1	I	0	0	0